Introdução a Teoria das Supercordas

1) Motivação
2) Partícula relativística → corda "bosonica"
3) Interações de cordas
4) Partícula com spin ½ → supercorda
5) Aplicações de supercordas

Eletromagnetismo

Relatividade Geral ? → Teoria Quântica de Campos contradição | Mec. Quântica ✓

1º ano de pós

"graduação"

IFT-UNESP - eu, Nastase, Mikhailov, Vieira
USP - Rivelles

→ Cordas

$\left(\frac{\partial}{\partial t} \quad \frac{\partial}{\partial x} \right)$

2º ano de pós

Polchinski, Vol 1, 2

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1) Aplicações para outra áreas de matemática e física

"generalização de teoria quântica de campos"

$\varphi(x)$ $\qquad\qquad$ $\varphi(x(\sigma))$

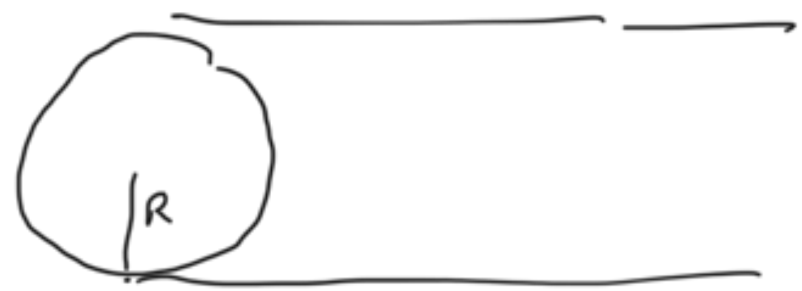
$(\vec{x}, t)$

$\sigma \in (0, 2\pi)$ "fechada"

$(t(\sigma=0) \;\; \vec{x}(\sigma=0))$

"aberta"

$\boxed{\text{Dualidades}}$



$$X_3 = X_3 + 2\pi R$$

Kaluza-Klein

Compactificação

$$\text{Teoria cordas } R = \text{Teoria cordas } \frac{\alpha'}{R}$$

constante "tensão"

$$\text{IIB } \lambda = \text{IIB } \frac{1}{\lambda}$$

AdS-CFT

Gravitação quântica

EM quântica: Qual a energia de um elétron

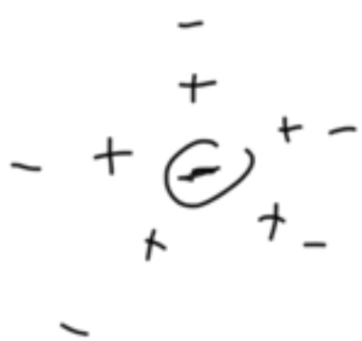
$$\text{Energia} = m_E c^2 + \text{energia campo elétrica}$$

$$+ 4\pi \int_{r_e}^{\infty} dr\, r^2 \frac{|E|^2}{4\pi} = e^2 \int_{r_e}^{\infty} dr \frac{1}{r^2}$$

$$- e^2/r \Big]_{r_e}^{\infty} \to \cancel{\infty}$$

$$- e^2/r_e$$

raio mínimo



$\boxed{10^{-10}\ \text{cm}}$

"renormalização"

Força forte — QCD

Força fraca — eletrofraca

EM → modelo "padrão" — TQC

$\hbar, G, c$

Planck energia

Plank cumprimento $\sim 10^{-33}$ cm

Energia $\quad m_E c^2 + \int_0 dr\, r^2 \left|\frac{m_E G}{r^2}\right|^2$

$$- \left.\frac{m_E^2 G^2}{r}\right)_0^\infty$$

m_E $\cancel{+ -}$ $\quad$ não existe "blindagem"

e singularidades não são eliminadas.

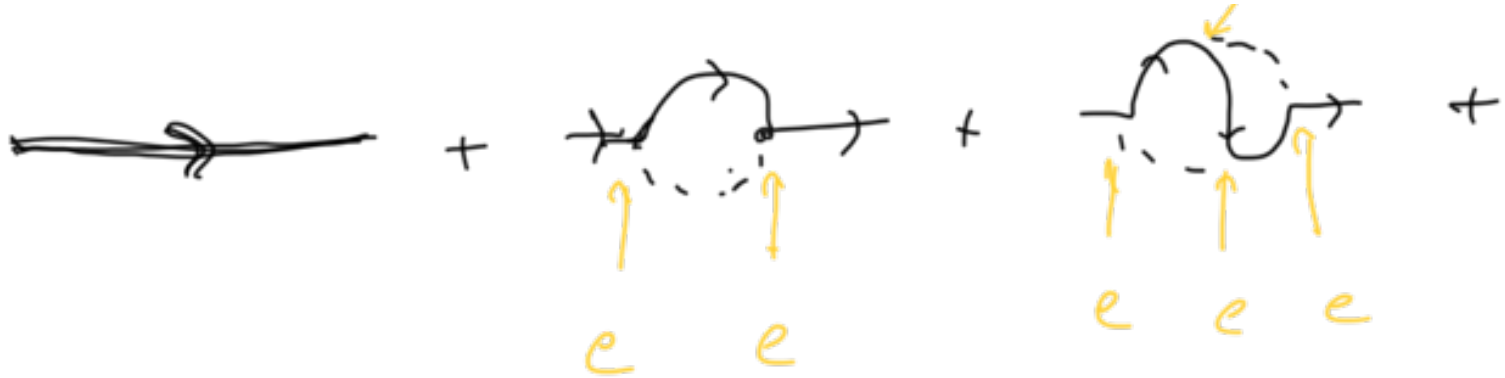
$$S = \int d^4x \left(F_{\mu\nu} F^{\mu\nu} + e A_\mu \bar{\Psi} \gamma^\mu \Psi + \bar{\Psi}(\not{\partial} - m)\Psi \right)$$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$\frac{1}{137}$ $\quad$ campo fóton $\quad$ campo eletron $\quad$ massa elétron

$$\left\langle e^{-S} \right\rangle$$

e



$$S = \int d^4x \, \underbrace{\sqrt{g}}_{\det g}\, R \qquad R = \partial\partial g$$

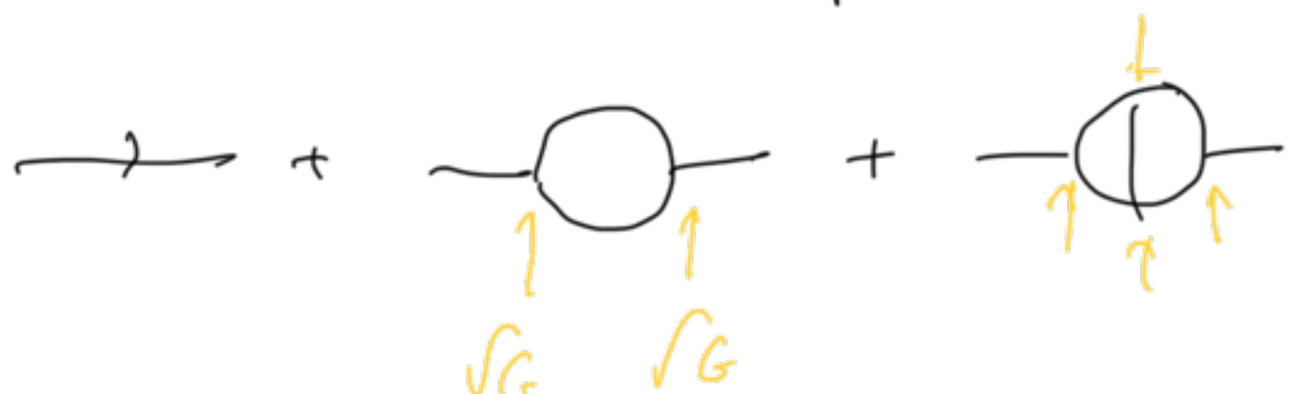
$g_{\mu\nu}(x)$

constante de Newton

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}\sqrt{G}$$

$$= \int d^4x \left[\partial h \partial h + \underbrace{\sqrt{G}\, h \partial h \partial h}_{\text{constante acoplamento}} + \dots \right]$$

$$G \to (\text{metro})^2$$

$\sim$ 

G carrega dimensões não-renormalizável

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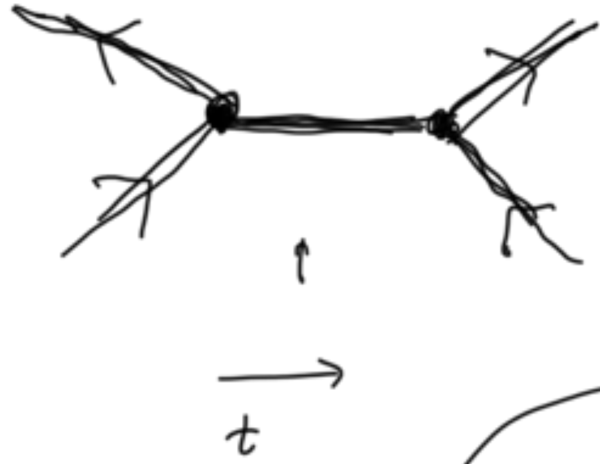
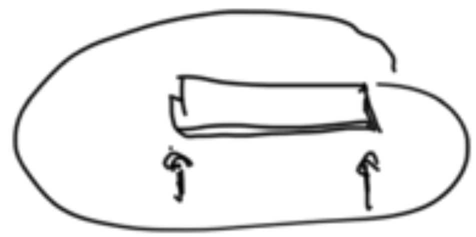
elektron
positron



1° quantizado

cordas



Na teoria das cordas, as partículas fundamentais

são vibrações diferentes da mesma corda

Interações não tem pontos singulares.

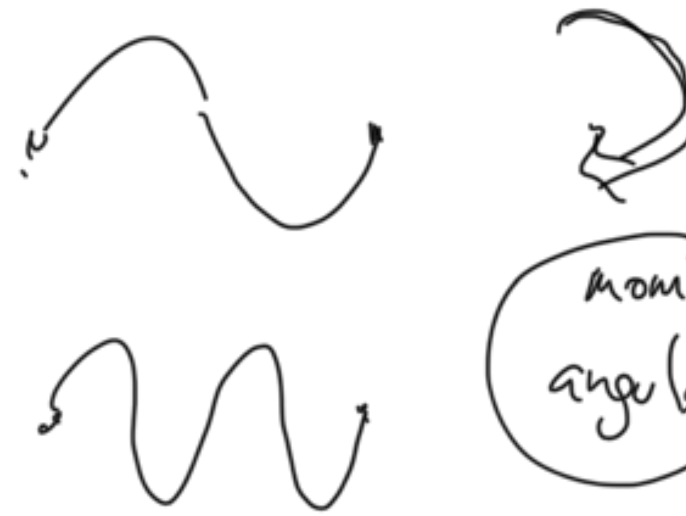
Membrana

Quarta:

espectro discreto

espectro contínuo

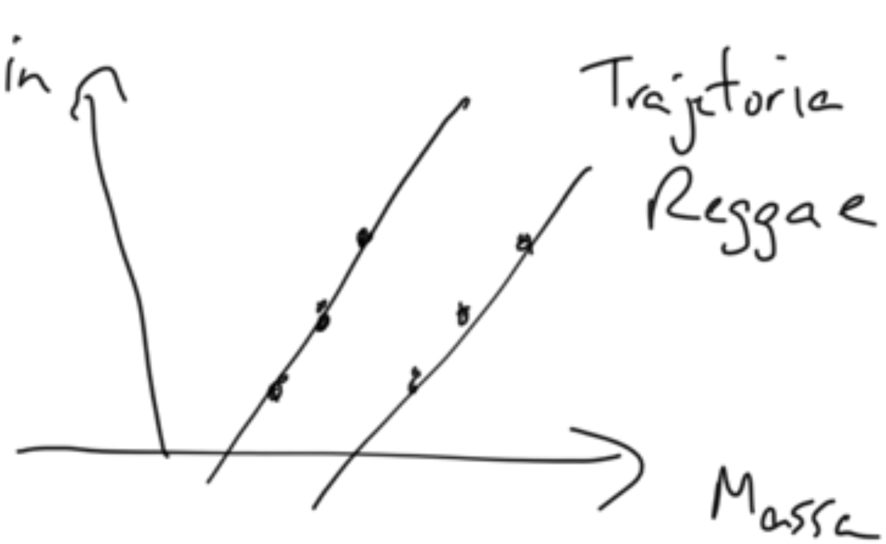
estados ligados de cordas



mom. angular

Teoria de cordas com férmions = "supercorda"

tem propriedade de "supersimetria"



Teoria dos Hadrons

Simetria

~~Lorentz~~

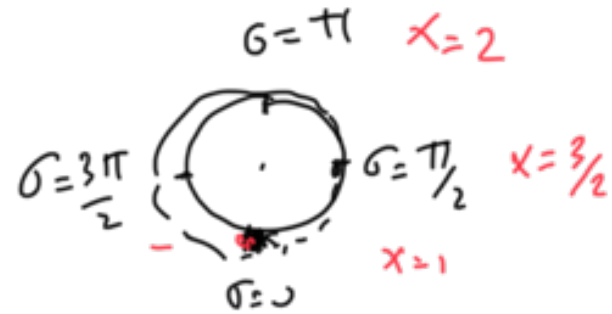
Poincaré

Translação

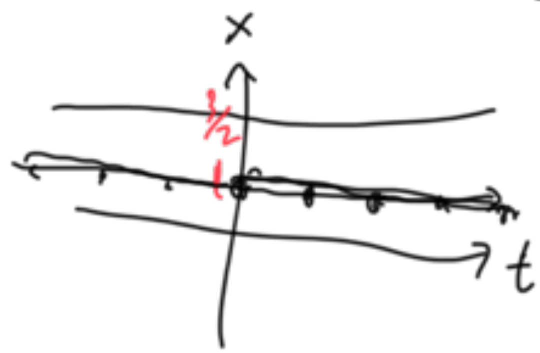
rotação

Supersimetria

bosons $\longleftrightarrow$ fermions



$x=1$



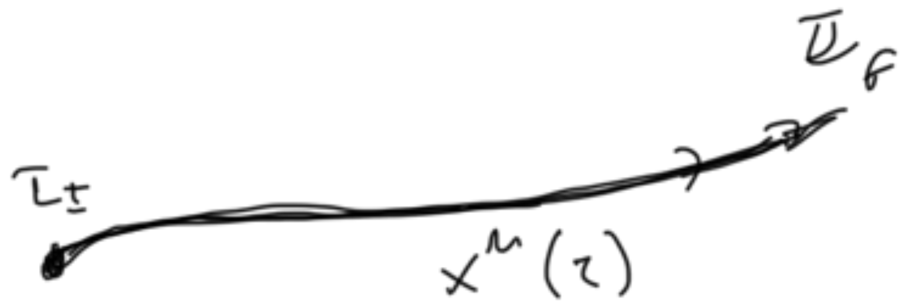
$t(\sigma=0)\ ,\ x(\sigma=0)=1$

t

x

Partícula relativística

$$x^\mu = (\underset{\substack{\| \\ ct}}{x^0}, x^1, x^2, x^3) \qquad \underset{\substack{\uparrow \\ \text{tempo próprio}}}{x^\mu(\tau)}$$



$$S = \int_{\tau_I}^{\tau_F} d\tau \, \mathcal{L} \left(x^\mu(\tau), \frac{\partial}{\partial \tau} x^\mu(\tau) \right)$$

$$\boxed{\frac{\partial \mathcal{L}}{\partial x^\mu(\tau)} = \frac{\partial}{\partial \tau} \left(\frac{\partial \mathcal{L}}{\partial \left(\frac{\partial x^\mu}{\partial \tau} \right)} \right)}$$

$$H = \frac{M}{2} \left(\frac{\partial x^i}{\partial \tau} \frac{\partial x^i}{\partial \tau} \right) = \mathcal{L}$$ quando energia potencial = 0

$$(-+++)$$

$$\boxed{\mathcal{L} = -Mc \sqrt{- \frac{\partial x^\mu}{\partial \tau} \frac{\partial x_\mu}{\partial \tau}}} \qquad \eta_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & +1 & & \\ & & +1 & \\ & & & +1 \end{pmatrix}$$

$$\frac{\partial x^0}{\partial \tau} \approx c \, , \quad \frac{\partial x^i}{\partial \tau} \approx v^i$$

$\overline{\partial \tau} \qquad , \quad \partial \tau$

$$\mathcal{L} = -Mc\sqrt{+c^2 - v^2} = -Mc^2\sqrt{1 - v^2/c^2}$$

$$= -Mc^2\left(1 - \tfrac{1}{2} v^2/c^2 + \dots\right)$$

$$= -\underline{Mc^2} + \underline{\tfrac{1}{2} M v^2 / \cancel{c^2}}$$

$c = 3 \times 10^8$ m/segundo

$$\text{comprimento da trajetória} = \int d\tau \sqrt{-\frac{dx^\mu}{d\tau}\frac{dx_\mu}{d\tau}} = \underline{\int dl} \quad \text{onde} \quad \underline{dl = \sqrt{-dx^\mu dx_\mu}}$$

$$|dx^\mu| = \sqrt{-dx^\mu dx_\mu}$$

$x^\mu(\tau)$

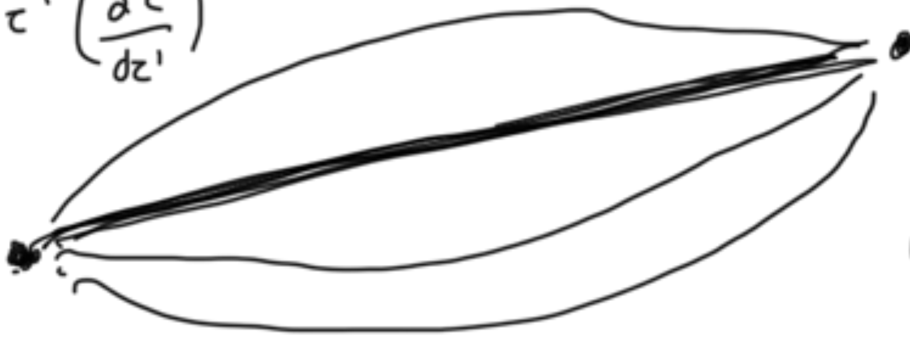
Eq. de mov

$\Rightarrow \mathcal{L}$ é minimizada

$\Rightarrow$ linha reta no caso livre

$\tau \to \tau'(\tau)$

$d\tau = d\tau' \left(\frac{\partial \tau}{\partial \tau'}\right)$



$$a = \left\langle \int e^{\frac{iS}{\hbar}} \right\rangle \qquad g_{\mu\nu}(x) = \eta_{\mu\nu}$$

$$\to \boxed{S = -Mc \int d\tau \sqrt{-\frac{\partial x^\mu}{\partial \tau}\frac{\partial x^\nu}{\partial \tau} g_{\mu\nu}(x)} + e\int d\tau\, A_\nu(x(\tau))\, \frac{\partial x^\nu}{\partial \tau}}$$

$$\frac{\partial L}{\partial x^\mu} = \frac{\partial}{\partial \tau}\left(\frac{\partial L}{\partial\left(\frac{\partial x^\mu}{\partial \tau}\right)}\right) \Rightarrow e\left(\frac{\partial}{\partial x^\mu} A_\nu(x)\right)\frac{\partial x^\nu}{\partial \tau} = +Mc\,\frac{\frac{\partial^2 x^\mu}{\partial \tau^2}}{\sqrt{-\frac{\partial x}{\partial \tau}\frac{\partial x}{\partial \tau}}} + e\frac{\partial}{\partial \tau}\left(A_\mu(x(\tau))\right)$$

$$\boxed{P_\mu = \frac{\partial L}{\partial\left(\frac{\partial x}{\partial \tau}\right)}} \qquad \frac{\partial L}{\partial x^\mu} = \frac{\partial}{\partial \tau} P_\mu$$

$$\to Mc\,\frac{\frac{\partial^2 x^\mu}{\partial \tau^2}}{\sqrt{-\frac{\partial x}{\partial \tau}\frac{\partial x}{\partial \tau}}} = e\left[\left(\frac{\partial}{\partial x^\mu} A_\nu\right)\frac{\partial x^\nu}{\partial \tau} - \frac{\partial}{\partial \tau}\left(A_\mu(x(\tau))\right)\right]$$

$$\frac{\partial}{\partial \tau}\left(A_\mu(x(\tau))\right) = \left(\frac{\partial}{\partial x^\nu} A_\nu\right)\left(\frac{\partial x^\nu}{\partial \tau}\right)$$

$\boxed{P_\mu = \frac{\partial x^\mu}{\cdots} Mc} \qquad \boxed{P_\mu P^\mu = M^2 c^2}$

$$+\boxed{F_\mu = \frac{\partial \tau}{\sqrt{-\frac{\partial x}{\partial\tau}\frac{\partial x}{\partial\tau}}}}$$

$$= e\, F_{\mu\nu}\frac{\partial x^\nu}{\partial \tau}$$

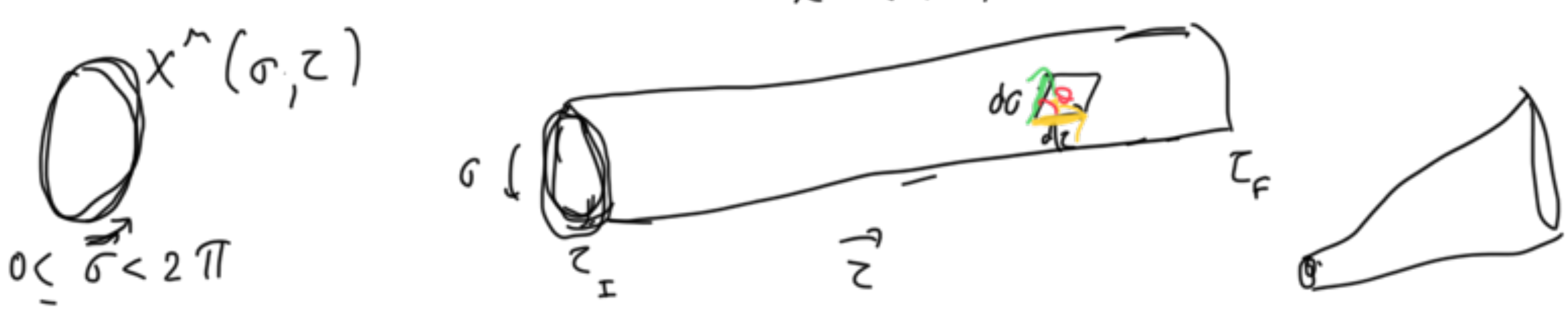
$$F_{\mu\nu} = \frac{\partial}{\partial x^\mu}A_\nu - \frac{\partial}{\partial x^\nu}A_\mu$$

$$\boxed{\mu = 1, 2, 3}$$

$$\frac{e}{c}F_{\mu\nu}\frac{\partial x^\nu}{\partial\tau} = \frac{e}{c}\left(F_{\mu 0}\, c + F_{\mu j}\, v^j\right) = \frac{e}{c}\left(E_\mu\, c + (B\times v)_\mu\right)$$

$$= e E_\mu + \frac{e}{c}(B\times v)_\mu$$

$x^\mu(\sigma, \tau)$



$$S = \int d\tau\, d\sigma \quad \left|\frac{dx}{d\tau}\right|\left|\frac{dx}{d\sigma}\right| \operatorname{sen}\theta$$

$$= \int d\tau\, d\sigma \quad \sqrt{\frac{dx}{d\tau}\cdot\frac{dx}{d\tau}}\sqrt{\frac{dx}{d\sigma}\frac{dx}{d\sigma}}\sqrt{1-\cos^2\theta}$$

$$\sqrt{1-\left(\frac{\frac{dx}{d\tau}\cdot\frac{dx}{d\sigma}}{\left|\frac{dx}{d\tau}\right|\left|\frac{dx}{d\sigma}\right|}\right)^2}$$

$$\rightarrow \quad S = T \int d\tau\, d\sigma \sqrt{\left(\frac{dx}{d\tau}\cdot\frac{dx}{d\tau}\right)\left(\frac{dx}{d\sigma}\cdot\frac{dx}{d\sigma}\right) - \left(\frac{dx}{d\tau}\cdot\frac{dx}{d\sigma}\right)^2}$$

tensão

$$\boxed{\begin{aligned}\tau &\to \tau'(\tau,\sigma)\\ \sigma &\to \sigma'(\tau,\sigma)\end{aligned}} \rightarrow \boxed{\begin{aligned}&\frac{\partial x}{\partial \tau}\cdot\frac{\partial x}{\partial \sigma} = 0,\\ &\frac{\partial x}{\partial \tau}\cdot\frac{\partial x}{\partial \tau} = -\frac{\partial x}{\partial \sigma}\cdot\frac{\partial x}{\partial \sigma}\end{aligned}}$$

$$\frac{\partial \mathcal{L}}{\partial x^\mu(\tau,\sigma)} = \frac{\partial}{\partial \tau}\left(\frac{\partial \mathcal{L}}{\partial\left(\frac{\partial x^\mu}{\partial \tau}\right)}\right) + \frac{\partial}{\partial \sigma}\left(\frac{\partial \mathcal{L}}{\partial\left(\frac{\partial x^\mu}{\partial \sigma}\right)}\right)$$

$$0 = T\frac{\partial}{\partial \tau}\left(\frac{\frac{\partial x}{\partial \tau}\left(\frac{dx}{d\sigma}\cdot\frac{dx}{d\sigma}\right) - \frac{dx}{d\sigma}\cancel{\left(\frac{dx}{d\tau}\cdot\frac{dx}{d\sigma}\right)}}{\sqrt{}}\right)$$

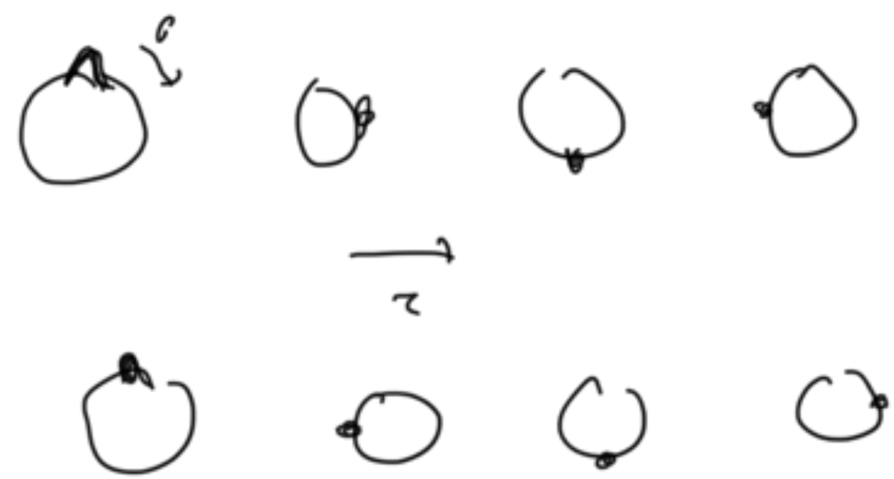
$$+\, T\frac{\partial}{\partial \sigma}\left(\frac{\frac{\partial x}{\partial \sigma}\left(\frac{\partial x}{\partial \tau}\cdot\frac{\partial x}{\partial \tau}\right) - \frac{\partial x}{\partial \tau}\cancel{\left(\frac{\partial x}{\partial \tau}\cdot\frac{\partial x}{\partial \sigma}\right)}}{\sqrt{}}\right)$$

$$\Rightarrow \quad 0 = T\left(\frac{\partial^2 X}{\partial \tau^2} - \frac{\partial^2 X}{\partial \sigma^2}\right)$$

$$\tau_+ = \tau + \sigma \quad , \quad \tau_- = \tau - \sigma$$

$$0 = T \frac{\partial}{\partial \tau_+} \frac{\partial}{\partial \tau_-} X^\mu(\tau, \sigma)$$

$$\Rightarrow \quad X^\mu(\tau, \sigma) = \underline{f^\mu(\tau_+) + g^\mu(\tau_-)}$$



$$a = \langle e^{iS} \rangle$$

$$\varphi(x) \quad \text{scalar} \rightarrow 2^a \text{ quantizado}$$

Particulas

$$x^\mu(\tau) \rightarrow 1^a \text{ quantizada}$$

$$P_\mu = \frac{\partial L}{\partial\left(\frac{\partial x^\mu}{\partial \tau}\right)}, \quad (P_\mu, x^\nu) = i\hbar \delta_\mu^\nu$$

Cordas

$$\varphi(x(\sigma)) \rightarrow 2^a \text{ quantizada}$$

$$P_\mu(\sigma) = \frac{\partial L}{\partial\left(\frac{\partial x^\mu(\sigma)}{\partial \tau}\right)} \qquad [P_\mu(\sigma), x^\nu(\sigma')] = i\hbar \delta_\mu^\nu \delta(\sigma - \sigma')$$

1^a quantizada

$$P_\mu = T \frac{\partial x}{\partial \tau}\left(\frac{\partial x}{\partial \sigma} \cdot \frac{\partial x}{\partial \sigma}\right) - \frac{\partial x}{\partial \sigma}\left(\frac{\partial x}{\partial \tau} \cdot \frac{\partial x}{\partial \sigma}\right)$$

$$P_\mu P^\mu = T^2 \left(\frac{\left(\frac{\partial x}{\partial \tau}\cdot\frac{\partial x}{\partial \tau}\right)\left(\frac{\partial x}{\partial \sigma}\cdot\frac{\partial x}{\partial \sigma}\right) - \left(\frac{\partial x}{\partial \tau}\cdot\frac{\partial x}{\partial \sigma}\right)^2 + \left(\frac{\partial x}{\partial \sigma}\cdot\frac{\partial x}{\partial \sigma}\right)\left(\frac{\partial x}{\partial \tau}\cdot\frac{\partial x}{\partial \tau}\right)}{\left(\sqrt{}\right)^2} \right)$$

$$= T^2 \, \underline{\underline{\frac{\partial x^\mu}{\partial \sigma} \frac{\partial x_\mu}{\partial \sigma}}}$$

proporcional a M^2

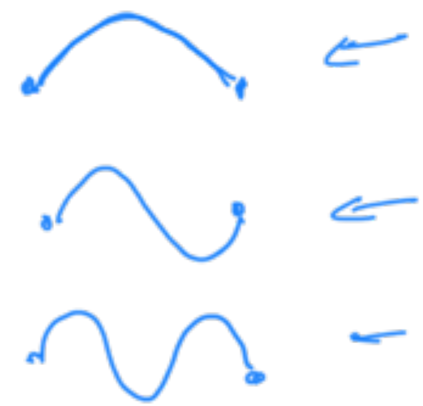
Espectro

$M^2 = 0$

$M^2 = T^2$

$M^2 = 2T^2$

$\vdots$



$$S = M \int d\tau \sqrt{\frac{\partial x}{\partial \tau}\cdot\frac{\partial x}{\partial \tau}}$$

$$= \int d\tau \left[e(\tau) \frac{\partial x}{} \cdot \frac{\partial x}{} + M^2 e^{-1}(\tau) \right]$$

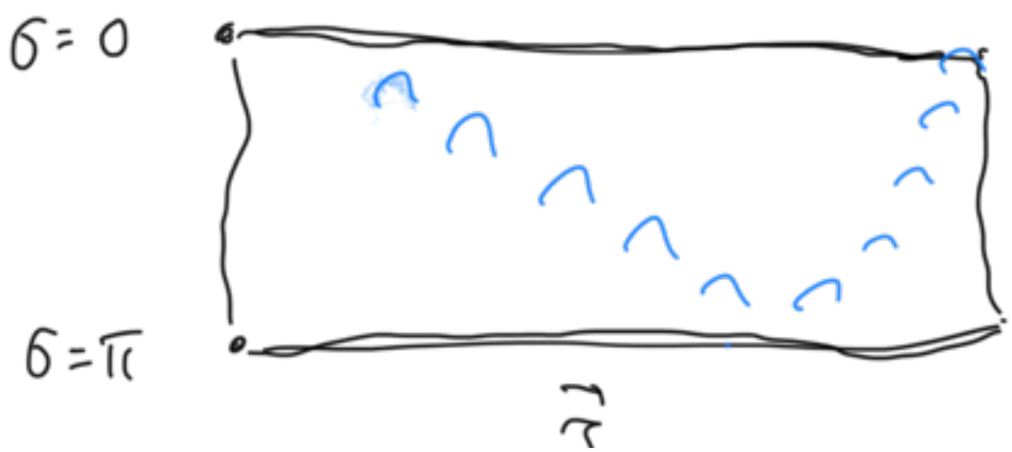
$$\frac{\partial f}{\partial e} = 0 \Rightarrow \frac{\partial X}{\partial \tau}\frac{\partial X}{\partial \tau} = M^{-2} e^{-2}$$

$$\Rightarrow e = M\left(\frac{\partial X}{\partial \tau}\frac{\partial X}{\partial \tau}\right)^{-1/2}$$

$$S = T\int d\tau\, d\sigma \left(h_{jk}\frac{\partial X}{\partial \tau_j}\cdot\frac{\partial X}{\partial \tau_k}\right)\sqrt{h}$$

$j,k = 1,2 \qquad \tau_1 = \tau$

$\tau_2 = \sigma$



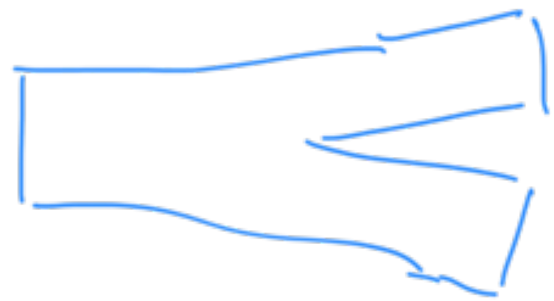
$\sigma = 0 \Rightarrow \frac{\partial X^\mu}{\partial \sigma} = 0$

$\sigma = \pi \Rightarrow \frac{\partial X^\mu}{\partial \sigma} = 0$

$$X^\mu = f(\tau+\sigma) + g(\tau-\sigma)$$

corda aberta $\rightarrow$ $A_{\mu}(x)$

corda fechada $\rightarrow$ $g_{\mu\nu}(x)$



$$S = M\int d\tau \underbrace{\sqrt{\frac{\partial X^\mu}{\partial\tau}\cdot\frac{\partial X^\nu}{\partial\tau}g_{\mu\nu}}}_{\text{comprimento}} \rightarrow S = M\int d\tau \frac{\partial X}{\partial\tau}\cdot\frac{\partial X}{\partial\tau}$$

$$+ e\int d\tau A_\mu \frac{\partial X^\mu}{\partial\tau}$$

$$\frac{\partial}{\partial\tau}\left(\frac{\partial X}{\partial\tau}\right) = 0$$

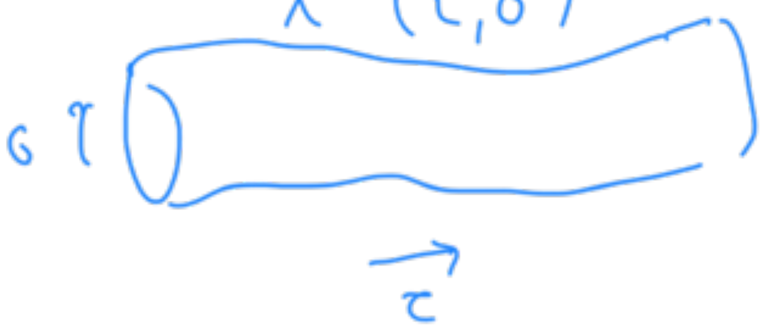
$X^\mu(\tau)$ $\qquad$ $X^\mu(\tau) = X_0^\mu + \tau p$

$$P_\mu = \frac{\partial L}{\partial\left(\frac{\partial X^\mu}{\partial \tau}\right)} = M \frac{\frac{\partial X_\mu}{\partial \tau}}{\sqrt{\frac{\partial X}{\partial \tau}\cdot\frac{\partial X}{\partial \tau}}} \Rightarrow \boxed{P_\mu P^\mu = M^2}$$

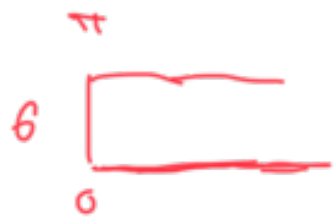
Eqn de Klein Gordons

$$S = T\int d\tau\, d\sigma \sqrt{\frac{\partial X}{\partial \tau}\cdot\frac{\partial X}{\partial \tau}\,\frac{\partial X}{\partial \sigma}\cdot\frac{\partial X}{\partial \sigma} - \left(\frac{\partial X}{\partial \tau}\cdot\frac{\partial X}{\partial \sigma}\right)^2}$$

$X^\mu(\tau,\sigma)$



$$\boxed{\begin{aligned} P_\mu P^\mu &= T^2 \frac{\partial X}{\partial \sigma}\cdot\frac{\partial X}{\partial \sigma} \\ P_\mu \cdot \frac{\partial X^\mu}{\partial \sigma} &= 0 \end{aligned}}$$



$$\to \quad S = T\int d\tau\, d\sigma\, g_{\mu\nu}\left(\frac{\partial X^\mu}{\partial \tau}\cdot\frac{\partial X^\nu}{\partial \tau} - \frac{\partial X^\mu}{\partial \sigma}\cdot\frac{\partial X^\nu}{\partial \sigma}\right) + \int_{\sigma=0} d\tau\, A_\mu \frac{\partial X^\mu}{\partial \tau}$$

$$= T\int d\tau_+\, d\tau_- \left(\frac{\partial X^\mu}{\partial \tau_+}\cdot\frac{\partial X^\nu}{\partial \tau_-}\right) g_{\mu\nu}(X) \qquad \tau_\pm = \tau \pm \sigma$$

$$\Rightarrow \frac{\partial}{\partial \tau_+}\frac{\partial}{\partial \tau_-} X^\mu = 0 \Rightarrow X^\mu = f(\tau_+) + g(\tau_-)$$

$$\underline{\text{Transf. conf.}} \begin{cases} \tau_+ \to \underline{j}(\tau_+) \\ \tau_- \to k(\tau_-) \end{cases} \qquad \tau_+ = \tau + i\sigma$$

$$\bar{\tau}_+ = \tau_- = \tau - i\sigma$$

$$\underline{\text{Inv. conf. quântica}} \Rightarrow g_{\mu\nu} \text{ satisfaz } 0 = \underline{R_{\mu\nu}} + \cancel{T^{-1}R^2} + T^{-2}\cancel{R^3} + \dots$$

τ

$$z = e^{i\theta} \qquad \theta \to \theta + c \Rightarrow z \to z e^{ic}$$

$$\underline{z \to z + c}$$

$$z \to j(z) \qquad z \to z^2$$

$$\boxed{D = 2} \qquad z, \bar{z} \qquad \tau, \sigma$$

$$\delta z \Rightarrow a + bz + cz^2 + dz^3 + \dots$$

$$D = 4 \qquad \delta x^\mu \Rightarrow \underset{4}{c^\mu} + \underset{6}{M^\mu{}_\nu x^\nu} + \underset{1}{D x^\mu} + (E^\nu x_\nu) x^\mu - (x \cdot x) E^\mu$$

$$\mu = 0, 1, 2, 3$$

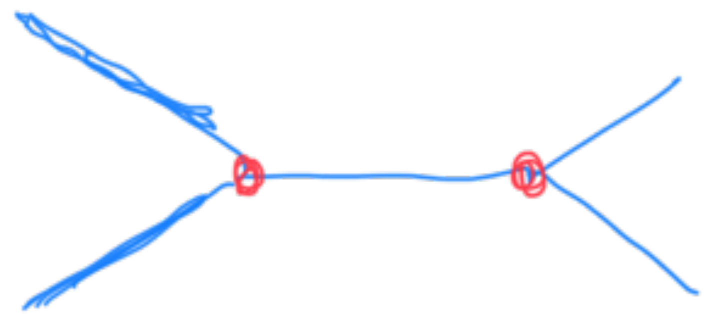
dilatação

boost conforme

$$m \to \frac{1}{D} m$$

1° quant.

$\to$?



$$S = M \int d\tau \left(\frac{\partial x}{\partial \tau} \cdot \frac{\partial x}{\partial \tau} \right) + \underline{\text{interação}}$$

infinito de interações possíveis

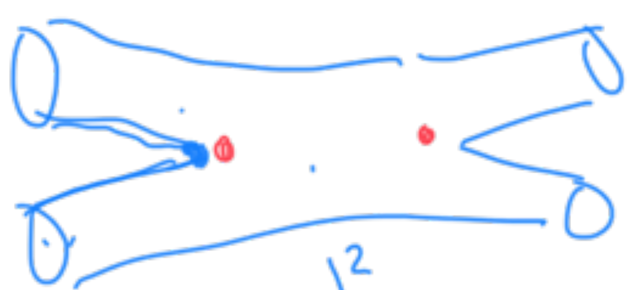
2° quant $$\tilde{S} = \int d^4x \left(\partial^\mu \varphi \partial_\mu \varphi + \lambda \varphi^3 \right)$$

constante de acoplamento

$$\varphi^4 , \ (\partial_\mu \varphi \partial^\mu \varphi) \varphi$$

$\to$

2



2

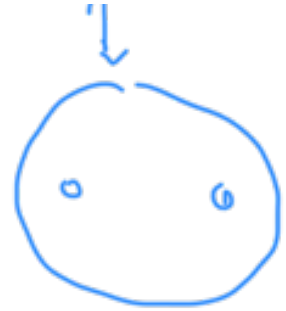
λ^2

1

1

$$\boxed{S = T \int d\tau\, d\sigma \left(\frac{\partial}{\partial \tau_+} X^\mu \cdot \frac{\partial}{\partial \tau_-} X^\nu g_{\mu\nu} \right)}$$

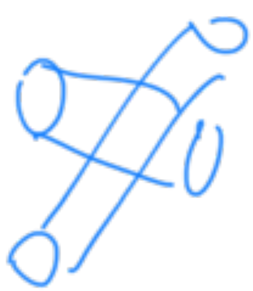
$$\boxed{\lambda^{2b+f-2}}$$



única interação possível

λ^4

λ^2

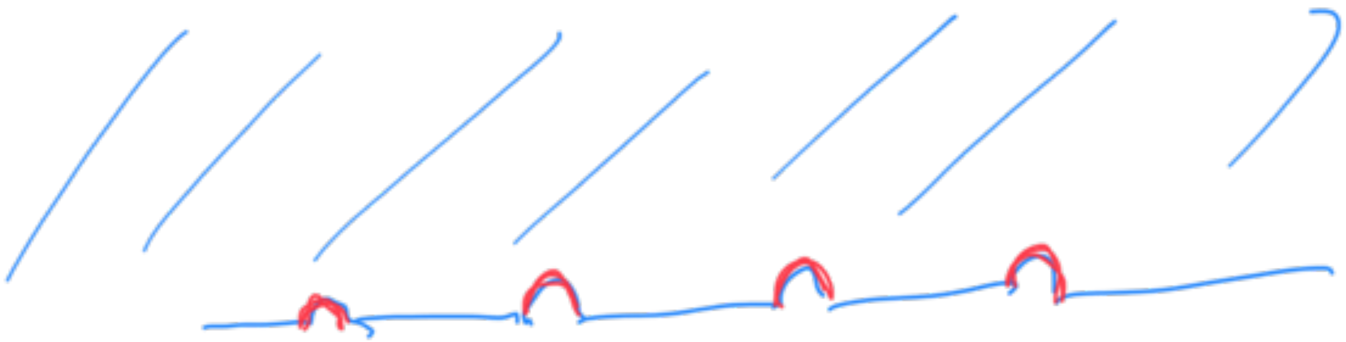


A_μ corda aberta

$g_{\mu\nu}$ corda fechada

$g_{\mu\nu} A^\mu A^\nu$

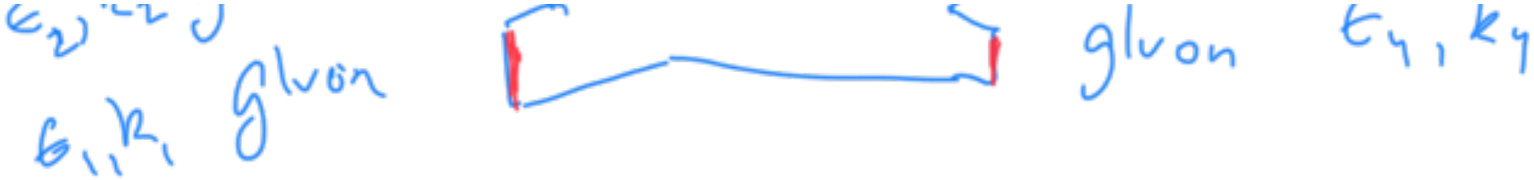
l_p

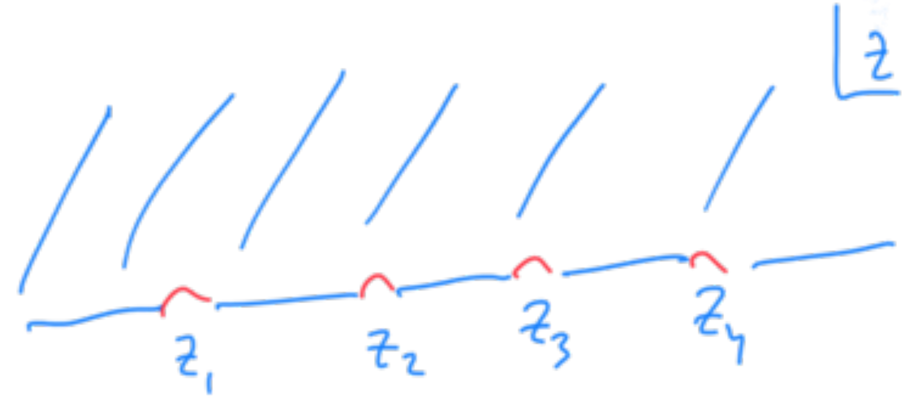


$$a = \langle e^{iS} \rangle$$

k_2 gluon

gluon ϵ_3, k_3





$$a_1 + a_2 = -a_3 - a_4$$

$$\boxed{a_1 + a_2 + a_3 + a_4 = 0}$$

$$\rho = a_1 \log(z - z_1) + a_2 \log(z - z_2) + a_3 \log(z - z_3) + a_4 \log(z - z_4)$$

$$A = \left\langle e^{iS} \, \epsilon_1^\mu \partial X_\mu(z_1) e^{ik_1 \cdot X(z_1)} \, \epsilon_2^\mu \partial X_\mu(z_2) e^{ik_2 \cdot X(z_2)} \cdots \right\rangle$$

$$\Box \varphi = \delta(\cancel{x} - \cancel{x}_0)$$

$$\varphi = G(\cancel{x}, \cancel{x}_0) = \boxed{\log(z - z_0)}$$

$$a = \epsilon_1 \cdot \epsilon_2 (\epsilon_3 \cdot k_1)(\epsilon_4 \cdot k_2) + \dots$$

$\boxed{\text{Corda bosônica}}$:

Espectro tem "taquions"

partículas com $m^2 < 0$

Espalhamento de gravitons tem singularidades

$\boxed{\text{Supercorda}}$

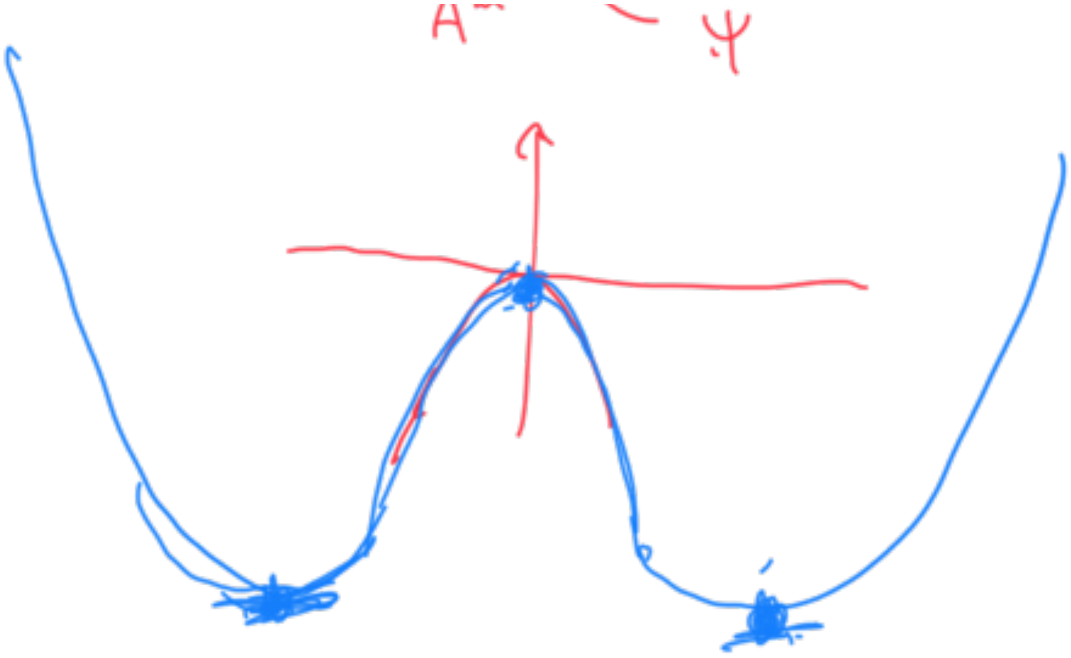
não tem taquions

tem férmions e bósons com "supersimetria"

$\boxed{\text{Geralização de partícula de spin } 1/2}$

$$\int d^4x \left(F_{mn} F^{mn} + \bar{\Psi}(\partial^m \gamma_m - M)\Psi + \bar{\Psi} \underline{A^m(x) \gamma_m} \Psi \right)$$

sinal ~~de~~ ausencia



$\to$ de teoria

$+ m^2 \varphi^2$

$(\varphi^2 - \mu^2)^2$

$\varphi(x)$

$X^\mu(\tau)$, $\psi^\mu(\tau)$

comutantes   anticomutantes

$[X^\mu, X^\nu] = 0$

$\{\psi^\mu, \psi^\nu\} = 0$

$$\frac{\partial \psi^\mu}{\partial \tau}\frac{\partial \psi_\mu}{\partial \tau} = -\frac{\partial \psi_\mu}{\partial \tau}\frac{\partial \psi^\mu}{\partial \tau}$$

$$\Rightarrow \frac{\partial \psi}{\partial \tau}\cdot\frac{\partial \psi}{\partial \tau} = 0$$

$$\to S = M\int d\tau \left(\frac{1}{2}\frac{\partial X}{\partial \tau}\cdot\frac{\partial X}{\partial \tau} + i\psi\cdot\frac{\partial \psi}{\partial \tau}\right)$$

$$\left(\psi\cdot\frac{\partial \psi}{\partial \tau}\right)^\dagger = \frac{\partial \psi^\dagger}{\partial \tau}\cdot\psi^\dagger$$

$$= \frac{\partial \psi}{\partial \tau}\cdot\psi$$

$$\boxed{P^\psi_\mu = \frac{\partial L}{\partial(\partial \psi)} = +iM\psi_\mu}$$

$$P^x_\mu = \frac{\partial L}{\partial(\partial x)} = 2M\frac{\partial X}{\partial \tau}$$

$$[P_\mu, x^\nu] = i\hbar\delta_\mu^\nu \Rightarrow \left[M\frac{\partial x^\mu}{\partial \tau}, x^\nu\right] = i\hbar\eta^{\mu\nu}$$

$$= -\psi\cdot\frac{\partial\psi}{\partial\tau}$$

$$\{p^\psi_\mu, \psi^\nu\} = i\hbar\delta_\mu^\nu \Rightarrow +iM\{\psi^\mu, \psi^\nu\} = i\hbar\eta^{\mu\nu}$$

$$\psi^3\psi^3 + \psi^3\psi^3 = \hbar/M$$

$$\boxed{\{\psi^\mu, \psi^\nu\} = \hbar/M\,\eta^{\mu\nu}}$$

$$\psi^\mu\psi^\nu = -\psi^\nu\psi^\mu$$

$$M^{\mu\nu} = \underbrace{x^\mu p^\nu - x^\nu p^\mu}_{\text{momento orbital}} \underbrace{-\psi^\mu p^{\psi\nu} + \psi^\nu p^{\psi\mu}}_{-2iM\psi^\mu\psi^\nu = -2i\tilde{\psi}^\mu\tilde{\psi}^\nu}$$

momento angular spin

Limite não-relativística $\Rightarrow \psi^0 = 0$
$(v \ll c)$

$$\left(\frac{\partial}{\partial x^\mu}\gamma^\mu\,\psi\right) = 0 \qquad \boxed{p^\mu\psi_\mu = 0}$$

$$\left(\begin{aligned} S^3 &= \psi^1\psi^2 = S^z \\ S^2 &= \psi^3\psi^1 = S^y \\ S^1 &= \psi^2\psi^3 = S^x \end{aligned}\right)$$

$$[S^x, S^y]$$

$$[\psi^2\psi^3, \psi^3\psi^1] = \psi^2\underbrace{\psi^3\psi^3}_{\hbar/2m}\psi^1 - \psi^3\psi^1\psi^2\psi^3$$

$$- \psi^1\psi^2\underbrace{\psi^3\psi^3}_{\hbar/2m}$$

$$= -\frac{\hbar}{m}\psi^1\psi^2 = -\frac{\hbar}{m}S^z$$

$$[S^i, S^j] = \epsilon^{ijk}S^k$$

spin $1/2$

$$\tilde{\psi} = \psi\sqrt{m}$$

$$\{\tilde{\psi}, \tilde{\psi}\} = m\{\psi, \psi\} = \hbar\,\eta^{\mu\nu}$$

$$\tilde{S} = \tilde{\psi}\tilde{\psi} \Rightarrow \{\tilde{S}^x, \tilde{S}^y\} = \hbar\,\tilde{S}^z$$

$$S = m\int dz\left(\frac{1}{2}\frac{\partial x}{\partial z}\cdot\frac{\partial x}{\partial z} - i\,\psi\cdot\frac{\partial\psi}{\partial z}\right)$$

$$+ e\int d\tau \left[A_\mu \frac{\partial x^\mu}{\partial \tau} + F_{\mu\nu}(x)\psi^\mu\psi^\nu \right]$$

$$M\frac{\partial^2 x^\mu}{\partial \tau^2} = eF^{\mu\nu}\frac{\partial x_\nu}{\partial \tau} + \partial^\mu F_{\rho\sigma}\psi^\rho\psi^\sigma$$

$$\rightarrow \quad \underline{M\frac{\partial \psi^\mu}{\partial \tau} = eF^{\mu\nu}\psi_\nu}$$

$$\frac{\partial}{\partial \tau}S^j = \underline{e(B\times S)^j}$$

$$S^j = \epsilon^{jkl}\psi^k\psi^l$$

$$\frac{\partial}{\partial \tau}S^j = \epsilon^{jkl}\frac{\partial}{\partial \tau}\psi^k\psi^l = e(B\times S)^j$$

Supersimetria relaciona $\underline{X^\mu}$, $\underline{\psi^\mu}$

$$\boxed{\begin{aligned}\delta X^\mu &= i\epsilon\,\psi^\mu \\ \delta\psi^\mu &= \epsilon\frac{\partial}{\partial \tau}X^\mu\end{aligned}}$$

simetria anticomutante

supersimetria em 1 dimensão

$$S = M\int d\tau \left(\frac{\partial x}{\partial \tau}\cdot\frac{\partial x}{\partial \tau} - i\psi\,\frac{\partial\psi}{\partial \tau} \right)$$

$$\delta S = M\int d\tau \left(2i\epsilon\,\frac{\partial\psi}{\partial \tau}\cdot\frac{\partial x}{\partial \tau} - i\epsilon\,\frac{\partial x^\mu}{\partial \tau}\cdot\frac{\partial \psi}{\partial \tau} - i\psi\,\frac{\partial}{\partial \tau}\left(\epsilon\,\frac{\partial}{\partial \tau}x^\mu\right)\right)$$

$$= M\int d\tau \left(2i\epsilon\,\frac{\partial\psi}{\partial \tau}\cdot\frac{\partial x}{\partial \tau} - i\epsilon\,\frac{\partial x}{\partial \tau}\cdot\frac{\partial \psi}{\partial \tau} + i\,\frac{\partial\psi}{\partial \tau}\cdot\epsilon\,\frac{\partial x}{\partial \tau}\right)$$

$$= M\int d\tau \left(2i\epsilon\,\frac{\partial\psi}{\partial \tau}\,\frac{\partial x}{\partial \tau} - 2i\epsilon\,\frac{\partial x}{\partial \tau}\cdot\frac{\partial \psi}{\partial \tau}\right) = 0$$

$$\cancel{\pm i\,\frac{\partial}{\partial \tau}\left(\psi\,\epsilon\,\frac{\partial}{\partial \tau}x^\mu\right)}$$

$$\boxed{P^2 = M^2}$$

$$\int_{\tau_I}^{\tau_F} d\tau\,\frac{\partial}{\partial \tau}\left(\psi^\mu\,\epsilon\,\frac{\partial}{\partial \tau}x_\mu\right)$$

$$= \epsilon\,\psi^\mu\,\frac{\partial}{\partial \tau}x_\mu\Big]_{\tau_I}^{\tau_F}$$

RNS

Supercorda (RNS)

$$\rightarrow S = \int d\tau_+ d\tau_- \left[\frac{\partial X}{\partial \tau_+} \cdot \frac{\partial X}{\partial \tau_-} + \psi^+ \cdot \frac{\partial}{\partial \tau_+} \psi^+ + \psi^- \cdot \frac{\partial}{\partial \tau_-} \psi^- \right]$$

$\tau_\pm = \tau \pm \sigma$ $\quad \frac{\partial}{\partial \tau_+}\frac{\partial}{\partial \tau_-} X^\mu = 0 \,, \quad \frac{\partial}{\partial \tau_+} \psi^+_\mu = 0 \,, \quad \frac{\partial}{\partial \tau_-} \psi^-_\mu = 0$

1ª quantizada $\quad X^\mu = f^\mu(\tau_+) + g^\mu(\tau_-) \,, \quad \psi^+_\mu(\tau_-) \,, \quad \psi^-_\mu(\tau_+)$

Supersimetria em 2 dimensões

Supersimetria em 4 dimensões

$$A_\mu(x) \,, \quad \varphi^\alpha(x)$$

2ª quant.

$$\rightarrow S = \int d^4x \left(F_{\mu\nu} F^{\mu\nu} + \bar{\varphi}^\alpha \gamma^\mu_{\alpha\beta} \partial_\mu \varphi \right)$$

Supersimetria em 11 dimensões

Supergravidade

$$S = \int d^{11}x \left(R + \ldots \right)$$

Supersimetria em 10 dimensões

2ª quantizada

$$S = \int d^{10}x \left(F_{\mu\nu} F^{\mu\nu} + \ldots \right)$$ super-YM

$$S = \int d^{10}x \left(R + \ldots \right)$$ supergravidade

Tipo I

Tipo IIA

Tipo IIB

Escondida na teoria usual

"espinores puros"

$$\Psi\left(x(\sigma) \right)$$

$\sigma = 0$

$$|\Psi(x(0))|^2$$

$$\mathcal{L} = \frac{1}{2} M \frac{\partial \vec{x}}{\partial t} \cdot \frac{\partial \vec{x}}{\partial t} + V(x, \Psi) + M \vec{\Psi} \cdot \frac{\partial \vec{\Psi}}{\partial t}$$

supercampo $\vec{X}(t, \kappa) = \vec{x}(t) + \kappa \vec{\Psi}(t)$

anticomutante

$$\{\kappa, \kappa\} = 0 \Rightarrow \kappa^2 = 0$$

$$D = \frac{\partial}{\partial \kappa} + \kappa \frac{\partial}{\partial t}$$

$$\tilde{\mathcal{L}} = \frac{1}{2} M D\vec{X} \cdot \frac{\partial X}{\partial t} + W(X)$$

$$S = \int dt \, d\kappa \, \tilde{\mathcal{L}}$$

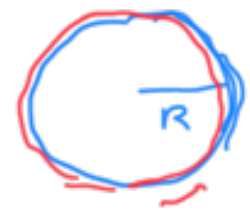
Dualidade

D=10 Teoria IIB supergravidade → supercorda IIB

Dualidade $\lambda \leftrightarrow \frac{1}{\lambda}$

const. de acoplamento

$-\infty < x_0, \dots, x_8 < \infty$ $\quad 0 \leq x_9 \leq 2\pi R$



x_0 = t

Supercorda neste espaço-tempo com raio R

=

Supercorda num espaço-tempo com raio $\frac{1}{2\pi R}$

---

1) Partículas neste espaço-tempo com raio R

$M = 0, \dots, 4$ $\quad g_{MN}$ $\quad \mu, \nu = 0, \dots, 3$

$g_{\mu\nu}$

$\tilde{g}_{\mu 4} = A_\mu$

$\tilde{g}_{44} = \varphi$

A_μ

$A_\mu^{(I)}$ Yang-Mills

$$\int d^5x \sqrt{g}\, R$$

$$= \int d^4x \left(\sqrt{\tilde{g}}\, \tilde{R} + \sqrt{\tilde{g}}\, F^2 + \dots \right)$$

$\mu = 0, \dots 3$

$$\varphi(x^\mu, y) = \varphi(x^\mu, y + 2\pi R)$$

$$y \simeq y + 2\pi R$$

$$\Rightarrow \varphi(x^\mu, y) = \sum_{m=-\infty}^{\infty} \tilde{\varphi}_m(x^\mu)\, e^{\frac{imy}{R}}$$

$$\boxed{P_M P^M \varphi = 0} \Rightarrow \left(P_\mu P^\mu + \frac{\partial}{\partial y}\frac{\partial}{\partial y} \right) \varphi(x, y) = 0$$

$$\left(P_\mu P^\mu - \left(\frac{m}{R}\right)^2 \right) \tilde{\varphi}_m = 0 \Rightarrow M_m \tilde{\varphi}_m = \frac{m^2}{R^2} \tilde{\varphi}_m$$

$$\left(P_\mu P^\mu - M^2 \right) \tilde{\varphi} = 0 \quad \text{and} \quad M = \frac{m}{R}$$

$(p_\mu p^\mu - M_m^2)\,\tilde\varphi_m = 0$ onde $M_m = \frac{m}{R}$

$R \to 0 \Rightarrow \tilde\varphi_0$ com massa zero

$\tilde\varphi_1, \tilde\varphi_{-1}, \tilde\varphi_2, \tilde\varphi_{-2}, \dots$ com massa $\to \infty$

Torre de Kaluza-Klein $\left\{ \begin{array}{l} \vdots \\ 3/R \\ 2/R \leftarrow \\ 1/R \leftarrow \\ 0 \end{array} \right.$

$$p^a = \int_0^{2\pi} d\sigma \left(\frac{\partial X^a}{\partial \tau}\right) M$$

$\varphi(x(\sigma))$

$x^a \simeq x^a + 2\pi R$

$\rightarrow$

$$p_\mu p^\mu = T^2 \frac{\partial X}{\partial \sigma} \cdot \frac{\partial X}{\partial \sigma}$$

$$0 = p_\mu p^\mu + p_a p_a$$

$X^9 = 2\sigma R$

$\frac{\partial X^9}{\partial \sigma} = 2R$

$\int_0^{2\pi} d\sigma \frac{\partial x^9}{\partial \sigma} = 2\pi R$

$2 \text{ voltas} = \int_0^{2\pi} d\sigma \frac{\partial x^9}{\partial \sigma} = 4\pi R$

n voltas

$$+ T^2 \frac{\partial X^4}{\partial \sigma} \cdot \frac{\partial X_r}{\partial \sigma} + T^2 \frac{\partial x^9}{\partial \sigma} \frac{\partial x^9}{\partial \sigma}$$

$(2\pi n R) \quad (2\pi n R)$

$$M^2 = P_9 P_9 + T^2 \int_0^{2\pi} \frac{\partial x^9}{\partial \sigma} \int_0^{2\pi} \frac{\partial x^9}{\partial \sigma}$$

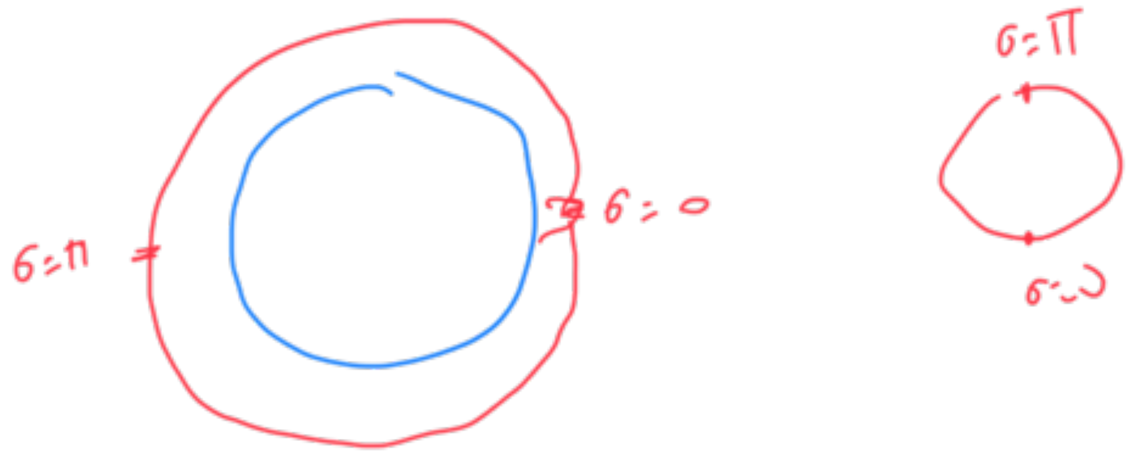
$$P_9 \sim \frac{m}{R} \qquad \int_0^{2\pi} \frac{\partial x^9}{\partial \sigma} d\sigma = 2\pi n R$$

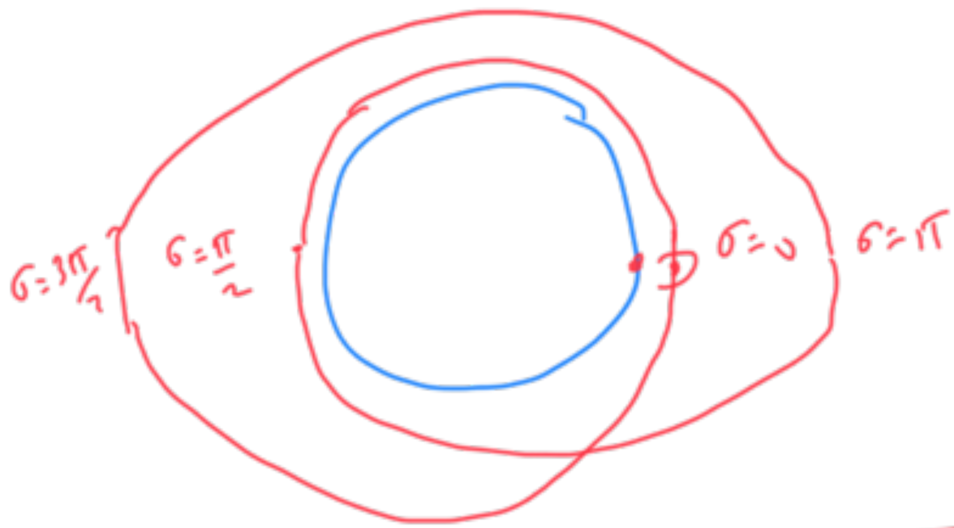
$$M^2 = \left(\frac{m}{R}\right)^2 + T^2 (2\pi n R)^2$$

$$= \left(\frac{m}{R}\right)^2 + n^2 \underline{(2\pi T R)}^2$$

Espectro Invariante se trocar $\quad m \longleftrightarrow n$

$\underline{R} \to 2\pi T R$





D=11 Teoria-M compactifica num círculo de raio R = IIA supercorda D=10 com

$\lambda \approx R^{2/3}$

Conjetura

Conjectura

$\uparrow$ constante de acoplamento

$$\Rightarrow \quad \text{IIB supercorda } D=10 \text{ com } \lambda$$

$$=$$

$$\text{IIB' supercorda } D=10 \text{ com } \frac{1}{\lambda}$$

$$\int d^{10}x \left(\sqrt{g}\, R + R^2 + R^3 \right)$$

GUT = "grand unified theory"

$$\underset{\substack{\uparrow \\ \text{em}}}{U(1)} \times \underset{\substack{\uparrow \\ \text{fraca}}}{SU(2)} \times \underset{\substack{\uparrow \\ \text{forte}}}{SU(3)} = \boxed{\begin{array}{c} SU(5) \\ \hline SO(10) \\ \hline E_6 \end{array}} \longleftarrow \text{Supercordas}$$

Anti-de-Sitter   Tem soluções com supersimetria

$-R$ const. cosmológico   raio R

$D=10$ $AdS_5 \times S^5$ $\longleftrightarrow$ $D=4$ super-Yang-Mills $\mathcal{N}=4$

com grupo de gauge $SU(N)$.

e const. de acoplamento g

$(gN) = R^4$

supercorda

AdS

CFT teoria de campos conforme

Não pode usar a supercorda RNS

Tem que usar a supercorda com espinores puros

Para calcular amplitudes multiloop

RNS : até 2 loops

Spinores puros : até 3 loops com supersimetria manifesta